NETWORK PERFORMANCE UNDER SYSTEM OPTIMAL AND USER EQUILIBRIUM DYNAMIC ASSIGNMENTS: IMPLICATIONS FOR ATIS

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This paper represents a comparative assessment of network cost and performance under time-dependent system optimal (SO) and user equilibrium (UE) assignment patterns, with particular reference to the effectiveness of Advanced Traveler Information Systems (ATIS). Both SO and UE solutions are found using a new simulation-based algorithm for the time-dependent assignment problem. Experiments are conducted using a test network with signal controlled junctions under progressively increasing network loading intensities. A diagnosis of system performance for various intensities of loading is effected using network-level traffic descriptors, for both system optimal and user equilibrium assignments.

The results affirm the validity of a meaningful demarcation between system optimal and user equilibrium assignments in urban traffic networks, and provide useful insights for macroscopic network-level relations among traffic descriptors. This suggests that ATIS information supply strategies based on system optimal route guidance could considerably outperform descriptive non-cooperative information strategies, especially at moderate to high congestion levels in the network. The results also illustrate the time-dependent nature of the gains achieved by a system optimal assignment vis-a-vis a user equilibrium assignment in a congested traffic network.

1. Introduction

1.1 Motivation and Problem Statement

Approaches incorporating advances in communication technologies, information processing systems, electronics and automation, broadly labeled as Intelligent Vehicle Highway Systems (IVHS), continue to generate considerable interest for their potential to alleviate urban and suburban congestion of traffic systems. Advanced Traveler Information Systems (ATIS) provide travelers with real-time information on existing traffic conditions and/or instructions on route selection from their current location to their destinations. Successful implementation of ATIS, especially at high market penetration levels, involves the dynamic assignment of vehicles to "optimal" paths to reduce overall system user costs. Recently, Mahmassani and Peeta [1, 2] proposed a heuristic algorithm to solve the system optimal dynamic traffic assignment problem for the ATIS context, where a central controller with known or predicted time-dependent origin-destination (O-D) trip desires over the horizon of interest solves for paths to prescribe to users in order to attain some system-wide objectives. A comprehensive review and discussion of dynamic assignment and traffic simulation models for ATIS/ATMS applications are given in Mahmassani et al. [3].

In this paper, we analyze the performance of a traffic network employing this solution methodology for both system optimal and user equilibrium time-dependent assignments. As in the static case, system optimal and user equilibrium dynamic assignments involve similar algorithmic steps, differing primarily in the specification of path travel costs that form the basis of the corresponding assignments. System optimal (SO) dynamic assignment is accomplished using time-dependent marginal travel times (see Ghali and Smith [4]), whereas a user equilibrium (UE) assignment is attained using the time-dependent average travel times. We analyze the system performance under the above assignment schemes for different intensities of network loading covering the spectrum of network states from uncongested networks to very highly congested networks. In addition, the numerical experiments illustrate the extent of the differences between SO and UE time-dependent assignments in terms of total system cost, at varying levels of network congestion. This question is of fundamental importance to ATIS operations, with regard to the relative benefits of normative versus descriptive information supply strategies.

A system optimal assignment does not generally represent an equilibrium flow pattern because some users may be able to obtain individual advantages simply by changing routes, though imposing a greater marginal cost to other users in the system in

the process. Its significance to the ATIS context lies in providing a benchmark against which other assignments or flow patterns can be gauged, thereby yielding an upper bound on the benefits attainable with real-time traffic information. A Wardrop UE holds when no user can improve his/her individual cost by unilateral route switching. There is no empirical evidence that UE conditions actually hold in real networks, though the UE solution is considered a reasonable and useful construct for the evaluation of long-term capacity improvements. Under real-time descriptive ATIS information on network conditions, a time-dependent UE pattern could be viewed as the result of the long-term evolution of the system, as users somehow learn and adjust under the supplied information. However, it is not at all clear that such convergence would be attained under inherently dynamic conditions (exacerbated by supplying information to users). Thus it is not known what the UE solution may represent from the standpoint of ATIS operation and evaluation. Actual user behavior and system performance under real-time descriptive information may be better or worse than the corresponding time-dependent UE solution in terms of the overall system cost. Nevertheless, a time-dependent UE pattern may be considered as a useful proxy for a favorable scenario of long-term network performance under real-time descriptive information.

It is known from static network equilibrium theory that SO and UE lead to identical solutions only for situations where the shortest paths taken by users are simultaneously the best paths from a system viewpoint. Such situations are observed when networks are relatively uncongested so that link operating speeds are unaffected by the flows on the links (limited vehicle interactions). At the other extreme, under very highly congested conditions, system performance is not likely to be markedly different under the two assignment schemes because the opportunities for SO to sufficiently ameliorate the traffic situation would probably be limited.

For network conditions between the two extremes, the extent of the differences between SO and UE solutions, particularly in terms of overall system cost, are not known. This is very important for ATIS, because if the two solutions are not perceptibly different, coordinated cooperative SO route guidance imposed by a central controller may not be necessary, and less complicated and simpler to implement descriptive information to non-cooperating drivers may be sufficient. If this were the case, there would be important implications for the focus that ATIS information supply strategies should take, with more attention directed to ways of guiding the system towards UE convergence and away from wide fluctuations. However, if SO indeed holds promise for meaningful gains over UE, then normative route guidance and/or strategies to induce the system near its SO

should be pursued. Of course, it is also desirable to ascertain network and traffic conditions under which differences between SO and UE are meaningful.

In this paper, overall user cost and network performance under time-dependent SO and UE assignment patterns are examined in a series of numerical experiments performed on a test network under different loading levels. The system performance is gauged using average network level traffic flow descriptors, in addition to the standard parameters like average travel time. The time-dependent nature of the problem further complicates the already intricate problem of characterizing traffic flow performance at the network level, previously addressed only under steady-state conditions, as discussed hereafter.

1.2   Network Traffic Flow Theory

Mahmassani, Williams and Herman [5, 6] generalized the definitions of speed, flow and concentration to the network level and examined their interrelation in their model of network traffic performance. These concepts are extended to the dynamic case in the current analysis, in order to characterize the vastly varying network traffic conditions (especially for medium to high network loading levels) during the peak period. Average network speed V (mph) is obtained as the ratio of total vehicle-miles to total vehicle-hours in the network over the duration of interest. The average network concentration K (vehicles per lane-mile), for the duration of interest, is the time average of the number of vehicles per unit lane-length in the system. However, the concentration varies dramatically with time in dynamic traffic networks. Hence, the time-dependent network concentration is examined by taking 5-min averages of number of vehicles per unit lane-length in the system. An overall measure of network concentration K over the duration of the period of interest is obtained by taking the arithmetic average of the 5-min averages. Similarly, time-dependent network flow, interpreted as the average number of vehicles per unit time that pass through a random point along the network, is examined by taking 5-min averages; an overall measure of network flow Q over the peak period is obtained by taking the simple average of $(\Sigma\ l_i\ q_i\ )\ /\ (\Sigma\ l_i\ )$, where q_i and l_i respectively denote the 5-min average flow and the length of link i, and the summations are taken over all network links.

Two fundamental relationships between these three network traffic flow variables are investigated in this study. The first relates average network speed, V, and average network concentration, K. For arterials or single roadways, a qualitative trend of decreasing speed with increasing concentration is well established. The same general trend was observed to hold at the network level in the simulation experiments of

Mahmassani et al. [6], though the complexity of network interactions preclude the analytic derivation of such a relation directly from the link-level relations. The second relationship analyzed is the basic identity Q = KV. Formally established for single roadways, it was shown to also hold at the network level in the previously mentioned steady-state experiments [6]. These experiments were performed keeping the network concentration level constant for the duration of interest by treating the network as a closed system. The NETSIM package was used for the study and vehicular behavior was governed by the comprehensive microscopic rules embedded in NETSIM. The present study replicates the network traffic conditions of a rush hour traffic situation. It uses the DYNASMART (DYnamic Network Assignment Simulation Model for Advanced Road Telematics) simulation-assignment model developed at The University of Texas at Austin for ATIS/ATMS applications. The Q = KV identity is expected to hold only approximately for time-varying network traffic flow.

This paper is organized as follows. The next section gives a brief summary of our solution methodology for the SO and UE problems. This is followed by a description of the experimental set-up, including the characteristics of the test network and traffic loading patterns. The results are then discussed, followed by concluding comments in the final section.

2. Solution Methodology

2.1 Problem Statement

Consider a traffic network represented by a directed graph G(N, A) where N is the set of nodes and A the set of directed arcs. A node can represent a trip origin, a destination and/or a junction of physical links. We consider a network with multiple origins and destinations. The time experienced by a vehicle to traverse a given link depends on the interactions taking place among vehicles in the traffic stream along this arc. The analysis period of interest, taken here as the peak period, is discretized into small equal intervals t = 1,........., T. Given a set of time-dependent O-D vehicle trip desires for the entire duration of the peak period, expressed as the number of vehicle trips r_{ijt} leaving node i for node j in time slice t, $\forall$ i, j $\in$ N and t = 1,........., T, determine a time-dependent assignment of vehicles to network paths and corresponding arcs. In other words, find the number of vehicles r_{ijt}^{k} that follow path k = 1,........., K_{ij} between i and j at time t, $\forall$ i, j $\in$ N and t = 1,........., T, as well as the associated numbers of vehicles on each arc l $\in$ A over time. As explained in the previous section, two such assignments are computed: 1)

one that satisfies UE conditions that no user can improve his actual (experienced) trip time by unilaterally changing routes, and 2) a SO assignment that minimizes total travel time (for all users) in the system over the peak period. The interpretation of these two solutions from the standpoint of ATIS effectiveness was discussed in the previous section.

2.2 Simulation-Assignment Solution Procedure

This section describes briefly the algorithm used to solve for SO and UE assignments. A detailed description of the solution procedure is provided in Mahmassani and Peeta [1, 2]. It consists of a heuristic iterative procedure in which a special-purpose traffic simulation model is used to represent the traffic interactions in the network, and thereby evaluate the performance of the system under a given assignment. As indicated earlier, the algorithmic steps for UE assignment are virtually identical to those for the SO solution except for the specification of the appropriate arc costs and the resulting path processing component of the methodology. The algorithm is first summarized for the SO case, followed by a brief description of the modification for the UE problem.

The use of a traffic simulation model to evaluate the SO objective function and model system performance circumvents the principal difficulties that have precluded solutions to realistic formulations of the problem, by obviating the need for link performance functions, link exit functions and implicitly ensuring that the first-in, first-out property holds on traffic facilities and that no unintended holding back of traffic takes place at nodes (see Mahmassani et al. [3] for a discussion of issues arising in dynamic traffic assignment). The algorithm uses the DYNASMART simulation-assignment model. DYNASMART has the capability to simulate the movement of individual vehicles through the network, with path selection decisions possible at every node or decision point along the way to the destination, as supplied by the user decision rules reflecting driver behavior in response to real-time information. In this work, vehicular paths are pre-assigned exogenously to DYNASMART, as determined by the steps of the SO or UE solution algorithms. Thus DYNASMART is used primarily as a simulator to replicate the dynamics of traffic phenomena in response to a given assignment of vehicles to paths. A detailed description of the various capabilities of DYNASMART is provided in Mahmassani et al. [7].

The simulation results provide the basis for a direction finding mechanism in the search process embodied in the solution algorithm for this nonlinear problem. The experienced vehicular trip times from current simulation are used to obtain a descent direction for the next iteration. The time-dependent shortest travel time paths and least marginal travel time paths are obtained using the time-dependent algorithms described in

Ziliaskopoulos and Mahmassani [8]. An elegant aspect of the solution methodology is that it avoids complete path enumeration between O-D pairs.

Flow Chart 1 depicts the solution algorithm for the system optimal dynamic traffic assignment problem. A brief summary of the approach is as follows:

1. Set the iteration counter I = 0. Obtain the time-dependent historical paths (paths obtained from database) for each assignment time step over the entire duration for which assignment is sought.

2. Assign the O-D desires (which are known a priori for the entire peak period) for the entire duration to the given paths and simulate the traffic patterns that results from the assignment using DYNASMART.

3. Compute the marginal travel times on links using time-dependent experienced or estimated link travel times and the number of vehicles on links obtained as post-simulation data (from step 2).

4. Using a special-purpose time-dependent least cost path algorithm, compute the least marginal time paths for each O-D pair for each assignment time step based on the marginal travel times obtained in step 3.

5. Perform an all-or-nothing assignment of O-D desires to the least marginal time paths computed in the previous step. The result is a set of auxiliary path vehicle numbers for each O-D pair for each assignment time step t = 1,............, T.

6. Update paths and the number of users assigned to those paths. Update of paths is done by checking if the path identified in step 4 already exists (i.e., has carried vehicles in at least one prior iteration) for that O-D pair and including it if it does not. The update of the number of vehicles (assignment of vehicles to the various paths currently defined between the O-D pair after the path update) is performed using the Method of Successive Averages (MSA), which takes a convex combination of the current path and corresponding auxiliary path numbers of vehicles, for each O-D pair and each time step. A detailed description of MSA is provided in Sheffi and Powell [9]. Note that other convex combination schemes could equally be used.

7. Check for convergence using an ∈-convergence criterion.

8. If convergence criterion is satisfied, stop the program. Otherwise, update the iteration counter I = I + 1 and go to step 2 with the updated data on paths and the number of vehicles assigned to each of those paths.

The complexity of the interactions captured by the simulator when evaluating the objective function generally preclude the kind of well-behaved properties required to guarantee convergence of the algorithm in all cases. However, such convergence was achieved in all the experiments reported in this paper, and in many other test networks

solved to date. Also, path marginals are not necessarily global as they are based on link level marginal travel times. Efforts were made to attain a global optimum where local solutions were suspected.

2.3 Modification to obtain User Equilibrium Solution

As previously discussed, the solution to the time-dependent UE problem is obtained by assigning vehicles to the shortest average travel time paths instead of the least marginal paths in the direction finding step (step 5). In other words, use the (time-dependent) average travel times on links instead of the marginal travel times in the shortest path calculations. In the above solution procedure, this simplifies step 3 and modifies step 4 as indicated.

3. Experimental Design and Set-up

This section first details the network configuration and traffic characteristics of the test network used in this study. This is followed by an illustration of the experimental set-up.

3.1 Network Configuration and Traffic Characteristics

The test network used in this study consists of a freeway with a street network on both sides as shown in Figure 1. It has 50 nodes and 163 links. Nodes within the freeway section are neither origin nor destination nodes. 38 origin nodes and 38 destination nodes are obtained by excluding freeway nodes (nodes 1-37 and 44). Freeway nodes are connected to the street network through entrance and exit ramps. Unless otherwise indicated in Figure 1, all arcs shown are two-directional. All links are 0.5 miles long and have two lanes in each direction except for the entrance and exit ramps which are directed arcs with a single lane. The freeway links have a mean free speed of 55 mph and the other links have a 30 mph mean free speed. In terms of traffic signal characteristics, 25 intersections have pre-timed signal control, 8 have actuated signal control and the remaining 17 nodes have no signal control.

3.2 Experimental Set-up

The comparative assessment of system performance for system optimal and user equilibrium assignments is conducted under different network loading levels, which generate different levels of network congestion. We define the network loading factor as the ratio of the total number of vehicles generated in the network during the assignment period to a given reference number (19403 vehicles over a 35-minute period in our experiments). Table 1 shows the different loading factors considered in this study, and the corresponding number of vehicles generated on the test network during the duration of

interest (35 minutes in all cases). In addition, it shows the corresponding number of "tagged" vehicles (vehicles generated for the 30 minute duration after the 5 minute start-up time) for which relevant performance statistics are accumulated. The loading factors range from 0.6 (very low congestion with 11616 vehicles) to 2.4 (extremely high congestion with 46674 vehicles). Under each loading level, the UE and SO solutions are obtained, and the resulting time-dependent link flow patterns are obtained from DYNASMART. Figure 2 shows a sample time-dependent loading pattern for a loading factor of 2.0. The indicated points on the graph correspond to the number of vehicles generated in the 5-minute interval centered on the location of each point; the lines connecting the points are physically meaningless and are included only for visual convenience. The shape of the loading curve for other network loading levels is approximately the same, though appropriately scaled in magnitude. This temporal pattern emulates real-world network loading for the peak period, with an initially increasing generation rate until a peak is reached, followed by a decreasing vehicle generation rate.

In the present study, a start-up time of 5 minutes is provided in DYNASMART for the network to be reasonably occupied, followed by a 30 minute peak period generation of traffic (for whom performance statistics are accumulated). Another aspect of the experimental set-up which critically influences the system performance is the spatial distribution of the O-D demand pattern. The vehicles generated are about evenly distributed spatially, both in terms of their origins and destinations, except for nodes 37 and 44 which generate/attract only about 25% the number of vehicles originating/destinated to a typical origin/destination node (ie, nodes 1-36).

4. Results

The results from the various experiments are viewed from two principal perspectives. First, they form the basis for comparison of system performance, particularly user costs under UE and SO assignment schemes, thereby addressing the questions relevant to ATIS information strategies described in the introductory section of the paper. Secondly, they are used to investigate network level traffic flow characteristics and relations using network-wide traffic descriptors. This investigation is conducted primarily for the SO flow pattern. An additional element of the study is the time-dependent analysis of the travel time gains of SO over UE, also of significance to ATIS operation.

The results provide several key insights from both of the above perspectives. They manifest a clear qualitative and quantitative distinction in the solution provided by the SO

assignment scheme as opposed to the time-dependent UE assignment procedure to route vehicles in a general traffic network. The results also reveal important and robust macroscopic relationships among network level traffic variables which parallel those for single roadways.

Table 2 reports summary statistics on the system performance for the SO assignment for the different loading factors. As expected, at low levels of network loading, when the network is relatively uncongested, the average travel times of vehicles in the network are relatively close across the different loading levels. As the load is increased, the effects of congestion become more prominent and the average travel times in the network increase at an increasing rate with the loading factor. At very high loading levels, the marginal effect of additional demand on system performance is very high. The results also indicate that there is only limited variation in the average distance traveled by vehicles under the various network loading levels, implying that greater congestion and not longer travel routes is the primary cause of the higher system trip times (the objective function seeks to minimize total system travel time only). Nevertheless, the average travel distance does increase with the loading level, reflecting an increasing percentage (though small in magnitude) of drivers assigned to longer travel routes.

Table 3 presents similar summary statistics for the UE assignment. The trends are similar to those described above for the SO case. The average travel distances under UE for various network loading levels are smaller than the corresponding distances for SO, indicating a smaller percentage of long travel routes under UE. This may be explained by some users being assigned to longer routes in order to reduce congestion elsewhere so as to reduce systemwide travel times.

Figure 3 shows comparatively the average trip times under various network loads for UE and SO assignments. As discussed above, both curves illustrate the increasing marginal effects of additional demand on system trip times. Of more relevance to the central question addressed in this paper, Figure 3 highlights the difference in the quality of the solutions provided by the two assignment rules for time-dependent network flows. This is further illustrated in Figure 4 which depicts the percentage improvement in average travel time of SO over UE (as a fraction of the UE travel time) for the various average network concentrations corresponding to the various levels of network loading. At low loading levels, SO and UE provide essentially identical solutions. For loading factors 0.6 and 0.8, SO shows improvements of 0.3% and 0.5% respectively over UE. At such low concentration levels, average link speeds remain relatively unchanged due to limited interactions among vehicles, and the marginal travel time on the link is essentially identical to the average travel time, leading to almost identical solutions under the two

assignment schemes. When network congestion increases slightly, to loading factors of 1.0 and 1.2, the corresponding SO trip time improvements are 3.0% and 4.5%, respectively, over the UE solution. As the network becomes moderately congested, system benefits under the SO assignment become more pronounced, with 10.6% and 11.2% improvements over UE for loading factors of 1.4 and 1.6 respectively. For heavily loaded networks, very substantial gains are obtained, with 15.1% and 19.0% improvements in system travel times using SO, for loading factors 1.8 and 2.0 respectively.

As the levels of network loading are increased further, the system reaches very high levels of congestion that near gridlock, and overall network throughput drops, making it increasingly difficult to discharge all vehicles from the system in a reasonable amount of time. Under these conditions, the ability to improve overall conditions by re-routing certain vehicles to paths with lower marginal costs diminishes, as all links become highly congested. Thus, the advantage of an SO assignment relative to UE begins decreasing, as reflected by reduced improvements of 12.4% and 10.7% for loading factors of 2.1 and 2.2 respectively. The gains begin dropping rapidly beyond this point, with higher loading levels eventually yielding negligible differences in the quality of the solution provided by the two schemes.

Figure 5 represents the average trip time improvement per vehicle under SO assignment for various levels of network loading. The results mirror the conclusions from Figures 3 and 4. Of course, this improvement in trip time is not experienced uniformly by all vehicles; in particular, it varies over the vehicle's time of departure during the peak period. The dynamic nature of the travel time savings is examined below.

Figure 6 depicts the cumulative demand generation as a function of time under the 2.0 loading factor along with the cumulative discharge curves under the SO and UE assignments. The various points on the plot are obtained by accumulating the statistics available for each 5-minute intervals. The area on the plot between the two discharge curves represents the time savings of SO over UE, in this case about 1438 hours. The figure illustrates the time-dependent nature of the benefits generated by SO over UE. When the network is in the early stages of loading (for about the first 20 minutes), it is not sufficiently congested to produce meaningful differences between SO and UE assignments. Most of the savings of SO are accrued between thirty and seventy minutes into the peak period as the network is close to peak congestion levels. Beyond seventy minutes, there appear to be virtually no significant gains of SO over UE as the network is again relatively uncongested. Thus the benefits of route guidance based on SO

assignment over UE routing are not accumulated uniformly over time — rather they are gained when the network is relatively well congested.

Figure 7 depicts the time savings per vehicle for SO over UE as a function of the vehicle's time of departure under different loading factors. To capture the time-dependency of the benefits in a systematic manner, travel time savings are accumulated based on the start times of the vehicles. In the figure, 0-5 on the y-axis (start time) refers to all vehicles that start between zero and five minutes. Vehicles that start during the first five minutes do not face congested conditions and hence SO does not yield savings over UE for these vehicles. Vehicles that start during the intervals 10-15 and 15-20 minutes accrue time savings at an increasing rate as the loading level increases. Over their trip, these vehicles encounter significant congestion that increases with the loading factor. For vehicles starting between 20 and 35 minutes, the benefits increase with network loading at an increasing rate until the 2.0 loading factor level, and then dip down. This trend illustrates the previously discussed tendency of diminished savings for SO under extremely high congestion conditions .

The time-varying nature of the savings of SO relative to UE and its dependence on the network load is further illustrated in Figure 8, which depicts two-dimensional plots of savings as a function of departure time, with each plot corresponding to a different loading factor. Figure 9 represents essentially similar information but in cumulative form. At a loading factor of 1.2, benefits are just perceptible for vehicles which enter the network during the latter half of the peak period as they face lightly congested conditions. A clearer picture emerges for a loading factor of 1.6 where the network is moderately congested for some duration. Vehicles departing in the first fifteen minutes do not encounter sufficient congestion in the network to obtain significant benefits for a SO assignment relative to UE. As congestion builds up, the SO assignment provides substantial benefits, until a peak is obtained for vehicles starting between twenty and twenty-five minutes. Vehicles generated beyond twenty-five minutes face decreasing levels of congestion as vehicles continue to discharge from the network. Hence, benefits begin diminishing for vehicles entering the network towards the end of the peak period. At a loading factor of 2.0, the same general trend is observed as above, though it is more marked because of the higher levels of congestion. Very high levels of congestion are observed for some period of time for a loading factor of 2.2, leading to reduced relative effectiveness of SO compared to UE for vehicles that face those congestion levels. This is reflected in the sudden drop of savings for vehicles starting between twenty and thirty minutes.

Network Flow Relations

The second aspect investigated through the experimental results relates to the macroscopic network level traffic theoretic relationships among network-wide traffic descriptors for dynamic traffic networks under consideration. The pertinent traffic variables and their averages over time and space were defined in the first section of the paper. As noted, while mathematical relationships among traffic flow variables are reasonably well established for arterials and intersections, the intricacies of interactions at the network level preclude analytic derivability of network-wide traffic relationships from the link-level traffic models. However, the simulation results extend the previous findings of Mahmassani et al. [5, 6] that the basic trends captured by the single roadway relationships seem to also hold at the network level for the dynamic case.

Figure 10 shows the average network speed and average trip time under different network loading levels for the SO assignment. Both curves are smooth indicating relatively robust performance characteristics at the network level, and clearly illustrating the increasing marginal effect of additional demand on the system performance.

The network level speed-concentration relationship for the SO assignment is depicted in Figure 11. Each point on the plot corresponds to a simulation run for the whole assignment period under a particular loading level. The figure clearly illustrates decreasing average network speed with increasing network concentration, paralleling the K-V relationship for an individual roadway. Note that the plot has a point of inflection corresponding approximately to the 1.8 loading factor. This qualitative trend has been observed previously in the simulation experiments of Mahmassani et al. [5] on a regular test network using the NETSIM package.

Table 4 examines the Q = KV relationship, which holds as an identity for a single roadway. Results indicate that Q and KV differ by less than 5% for all cases, which is well within the error introduced by the manner in which the time averages were computed. As described in the first section, the average network flow and concentration were calculated as an overall average of 5-minute averages, whereas the average network speed was determined through quantities accumulated every 0.1 minute (length of a simulation interval) of the simulation.

Figures 12 and 13 represent the network flow-concentration and speed-flow relationships respectively. The plots indicate that the Q-K and V-Q relationships parallel those for single roadways up to moderate levels of congestion, diverge for highly congested network and become confluent for very high congestion levels.

An essential element to be noted in the network level analysis is the time-dependent nature of the phenomena of interest. Averaging quantities like network flow

and concentration over the duration of the peak period is likely to mask the time-dependency of network performance. For example, overall network concentration is obtained by averaging low levels of concentration at both ends of the peak period and high levels in between, as shown in Figure 14 which depicts the time-dependent variation of concentration (normalized by dividing by a jam concentration of 160 veh./lane-mile) over the duration of interest. More detailed investigation of the interrelationships among network level traffic descriptors over time will be reported elsewhere.

5.   Concluding Comments

The experiments performed using the simulation-based algorithm to solve both the SO and UE versions of the time-dependent traffic assignment problem have provided insights of critical importance to the design of ATIS information supply strategies and results of fundamental significance in the context of network assignment and network traffic flow theories. Of course, experimental results from a single test network preclude definitive generalizations; nevertheless, they proffer an illustration of the insights that can be obtained on the basic constitution of the problems being addressed while suggesting directions for future research. The first main conclusion is that the results suggest meaningful differences in overall system cost and performance between time-dependent system optimal and user equilibrium assignments. The second main conclusion is that traffic networks under time-dependent traffic assignment patterns continue to operate within the envelope of relatively simple network traffic flow relationships that exhibit strong similarities to the traffic models established for individual road sections.

If we take the UE assignment results as somehow indicative of the situation that might be attained over time in a system where drivers have access to real-time on-board descriptive information through ATIS, the results of our experiments suggest that there is considerable potential for system optimal, coordinated route guidance, especially in heavily congested (though not oversaturated) networks. These results appear to contradict unsupported claims that descriptive information would likely perform as well as normative SO route guidance because UE system costs were claimed to be very close to SO costs. Instead, they strengthen previous recommendations (e.g., in Mahmassani and Jayakrishnan [10]) that coordinated information is necessary beyond a certain market penetration level.

The results further highlight the dynamic nature of the benefits accumulated by a SO assignment over UE. They suggest that SO is most effective when the traffic network is moderately to highly congested. In the context of peak period traffic, this implies that

most savings through SO assignment would be achieved not at the beginning nor end of the peak period, but in a time range in between. When the network is lightly or very highly congested (oversaturated), an SO assignment does not perform significantly better than UE. For relatively uncongested traffic situations, SO and UE yield almost identical solutions.

Our future research on this topic will investigate the system performance under partial user compliance when users are provided with "system optimal" paths, thereby introducing an additional element of user behavior. With regard to the traffic network flow theoretic aspects, avenues for future efforts in this area include in analyzing dynamic traffic networks from the perspective of the two-fluid theory of town traffic developed by Herman and Prigogine [11].

In conclusion, it is possible to characterize traffic flow in urban traffic systems using relatively simple macroscopic relationships, which parallel traffic flow relationships at the individual roadway level. It should be emphasized that simulation is an abstract representation of real-world traffic, and thus the research is mostly exploratory rather than definitive in nature. Results to date strongly suggest that the performance of dynamic traffic networks is critically sensitive to network topology and network loading pattern.

Acknowledgements

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                                                         ┌─────────────────────┐
                                                         │        I=0          │
                              ┌──────────────────┐       │  O-D DESIRES AND    │
                         ┌───►│   XP(O,D,T,K,I)   │◄──────│  HISTORICAL PATHS   │
                         │    └──────────────────┘       └─────────────────────┘
                         │              │
                         │              ▼
                         │    ┌──────────────────┐
                         │    │       PATH        │
                         │    │    ASSIGNMENT     │
                         │    └──────────────────┘
                         │              │
                         │              ▼
                         │    ┌──────────────────┐
                         │    │     DYNASMART     │
                         │    └──────────────────┘
                         │              │
                         │              ▼
                         │    ┌──────────────────┐
                         │    │  LINK MARGINAL    │
                         │    │  TRAVEL TIMES     │
                         │    └──────────────────┘
              ┌────────┐ │              │
              │ I=I+1  │ │              ▼
              └────────┘ │    ┌──────────────────┐
                   ▲     │    │  TIME-DEPENDENT   │
                   │     │    │  LEAST COST PATH  │
                   │     │    │    ALGORITHM      │
                   │     │    └──────────────────┘
                   │     │              │
                   │     │              ▼
                   │     │    ┌──────────────────┐
                   │     │    │   ALL-OR-NOTHING  │
                   │     │    │    ASSIGNMENT     │
                   │     │    └──────────────────┘
                   │     │              │
                   │     │              ▼
                   │     │    ┌──────────────────┐
                   │     │    │    AUXILIARY      │
                   │     │    │      PATHS        │
                   │     │    │  YP(O,D,T,K,I)    │
                   │     │    └──────────────────┘
                   │     │              │
                   │     │              ▼
                   │     │    ┌──────────────────┐
                   │     │    │     UPDATE        │
                   │     │    │      (MSA)        │
                   │     │    └──────────────────┘
                   │     │              │
                   │     │              ▼
                   │     │          ◇────────◇         ┌────────┐
                   │     │         ╱ CONVERGE ╲──YES──►│  STOP  │
                   │     │         ╲          ╱         └────────┘
                   │     │          ◇────────◇
                   │     │              │
                   │     │             NO
                   │     └──────────────┘
                   └────────────────────┘
```

XP(O,D,T,K,I) -- The number of O-D desires in period T assigned to path K between origin O and destination D at the I th iteration

YP(O,D,T,K,I) -- All O-D desires in period T are assigned to auxiliary path K between origin O and destination D at the I th iteration

Method of Successive Averages (MSA)
$$[XP(O,D,T,K,I+1) =$$
$$(1-\alpha) * XP(O,D,T,K,I) +$$
$$\alpha * YP(O,D,T,K,I)]$$
where
$$\alpha = 1/(I+1)$$
$$I = 0,1,2,...............$$

Flow Chart 1. Solution Algorithm for the System Optimal
Dynamic Assignment Problem

TABLE 1. Loading Factors and the Corresponding Numbers of Generated Vehicles and Tagged Vehicles for the Numerical Experiments

Loading Factor	No. of Generated Vehicles	Tagged Vehicles
0.6	11616	10585
0.8	15509	14098
1.0	19403	17621
1.2	23305	21145
1.4	27196	24697
1.6	31090	28205
1.8	34978	31726
2.0	38871	35258
2.1	40818	37014
2.2	42769	38784
2.4	46674	42322

TABLE 2.   Summary Statistics for System Optimal Assignment

Loading Factor	Av. Trip Time	Total Trip Time	Average Trip Distance	Total Trip Distance	Average Speed
	(minutes)	(hours)	(miles)	(miles)	(mph)
0.60	3.85	679.54	1.82	19257.75	28.34
0.80	3.90	916.05	1.81	25447.00	27.78
1.00	4.03	1183.06	1.82	22092.25	27.13
1.20	4.40	1549.48	1.84	38837.25	25.06
1.40	4.86	1999.10	1.85	45724.75	22.87
1.60	6.04	2837.07	1.92	54133.25	19.08
1.80	7.65	4042.91	1.97	62398.50	15.43
2.00	10.46	6149.46	1.99	70208.00	11.42
2.10	13.08	8071.91	2.01	74398.00	9.22
2.20	16.57	10710.93	1.99	77287.00	7.22
2.40	24.95	17601.78	2.13	89987.00	5.11

TABLE 3. Summary Statistics for User Equilibrium Assignment

Loading Factor	Av. Trip Time	Total Trip Time	Average Trip Distance	Total Trip Distance	Average Speed
	(minutes)	(hours)	(miles)	(miles)	(mph)
0.60	3.86	681.52	1.80	19103.75	28.03
0.80	3.92	920.81	1.78	25139.00	27.30
1.00	4.15	1219.46	1.79	31593.75	25.91
1.20	4.60	1622.47	1.81	38238.75	23.57
1.40	5.43	2236.52	1.80	44573.75	19.93
1.60	6.79	3192.16	1.85	52299.25	16.38
1.80	9.00	4762.95	1.88	59708.00	12.53
2.00	12.91	7587.70	1.96	69149.50	9.11
2.10	14.94	9215.69	1.93	71479.50	7.76
2.20	18.55	11993.56	1.99	77163.00	6.43

TABLE 4 Results of the Q, KV Comparison

LF	K veh/lane-mile	V miles/hr	KV veh/lane-hr	Q veh/lane-hr	% Difference (KV-Q)/Q
0.6	6.31	28.34	178.83	173.41	3.12
0.8	8.35	27.78	231.96	224.56	3.30
1.0	10.63	27.13	288.39	278.42	3.58
1.2	14.02	25.06	351.34	337.94	3.97
1.4	17.30	22.87	395.65	378.70	4.48
1.6	22.37	19.08	426.82	408.78	4.41
1.8	23.94	15.43	369.39	357.29	3.39
2.0	30.32	11.42	346.25	336.99	2.75
2.1	34.82	9.22	321.04	315.42	1.78
2.2	42.95	7.22	310.10	303.12	2.30
2.4	52.91	5.11	270.37	273.65	-1.20
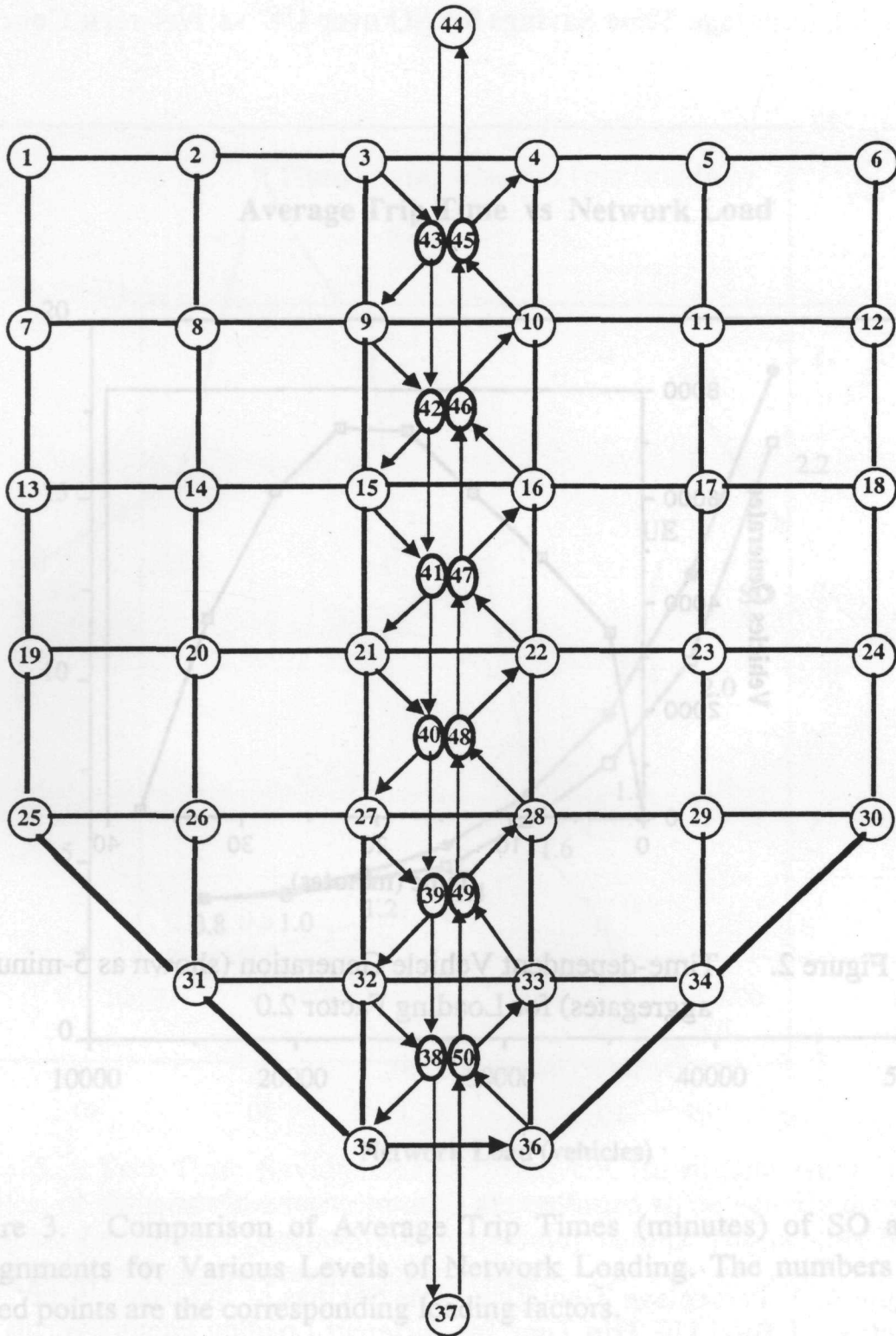
Figure 1. Network Structure

Figure 2.   Time-dependent Vehicle Generation (shown as 5-minute aggregates) for Loading Factor 2.0
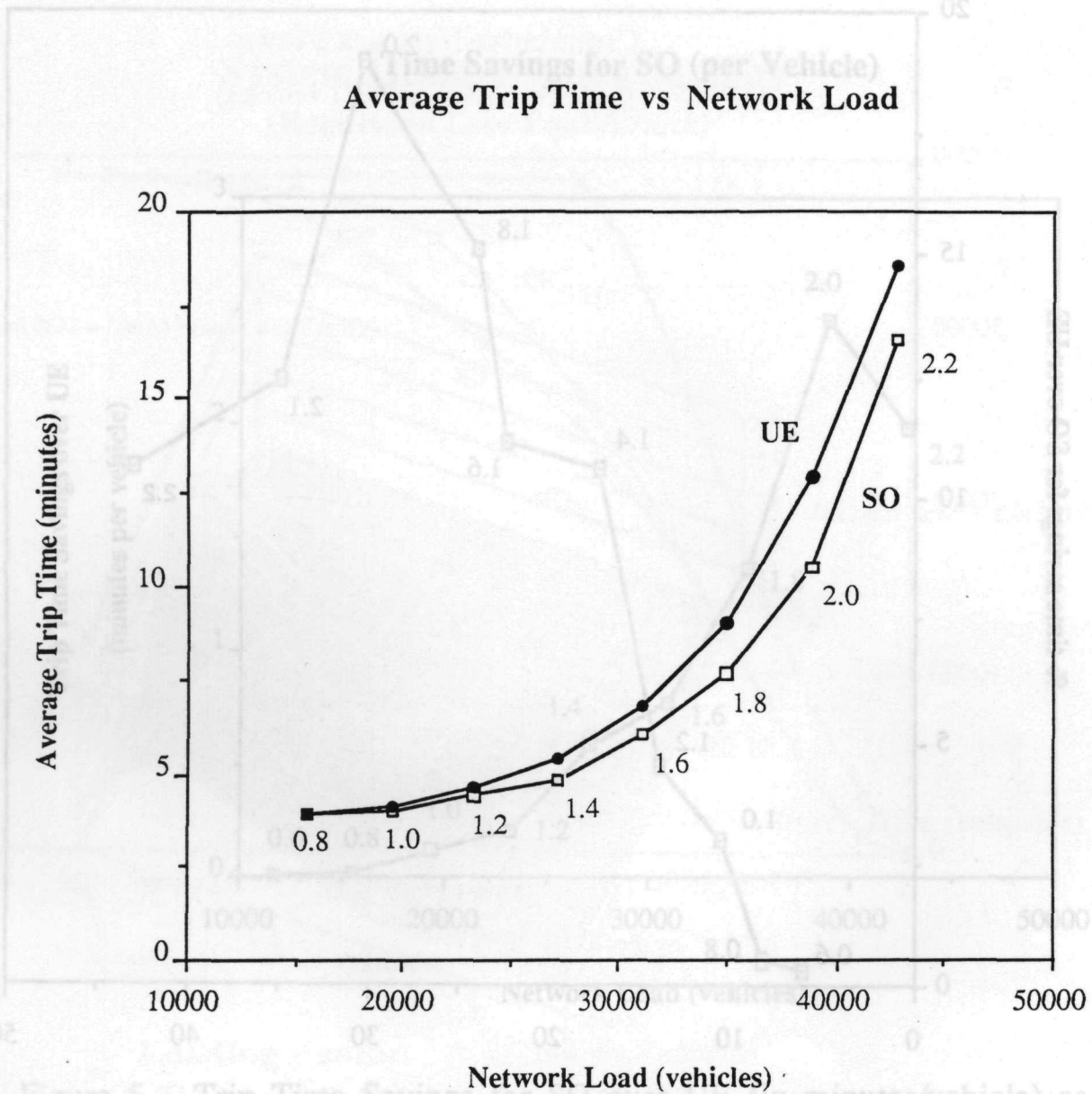
Average Trip Time vs Network Load

Figure 3. Comparison of Average Trip Times (minutes) of SO and UE Assignments for Various Levels of Network Loading. The numbers by the plotted points are the corresponding loading factors.

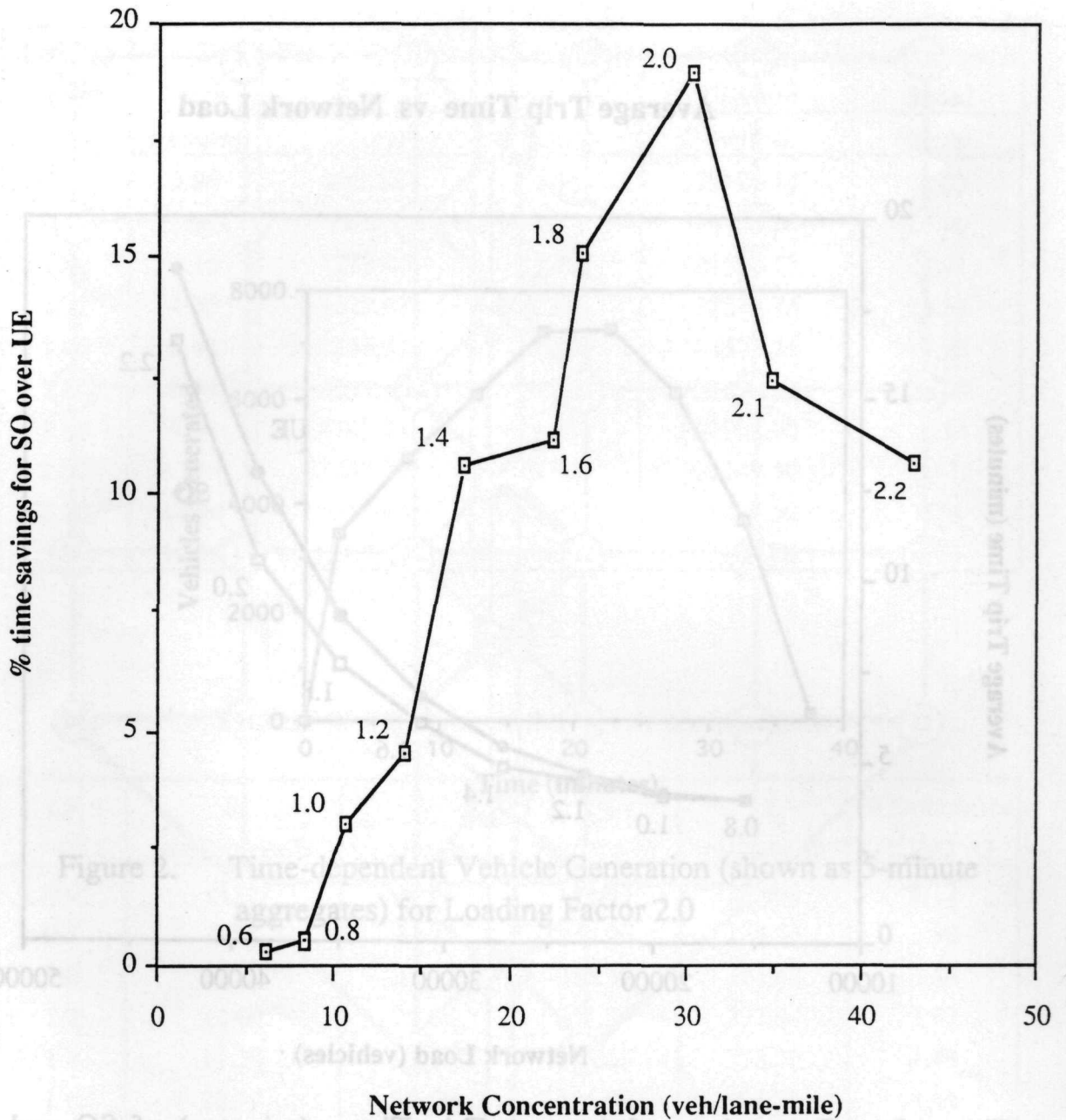# Percentage Time Savings for SO over UE vs Network Concentration

Network Concentration (veh/lane-mile)

Figure 4.   Percentage Total Trip Time Savings of SO over UE obtained as a Fraction of Total UE Trip Time for Different Loading Factors versus Average Network Concentration. The number by each plotted point is the corresponding loading factor.

Time Savings for SO (per Vehicle)
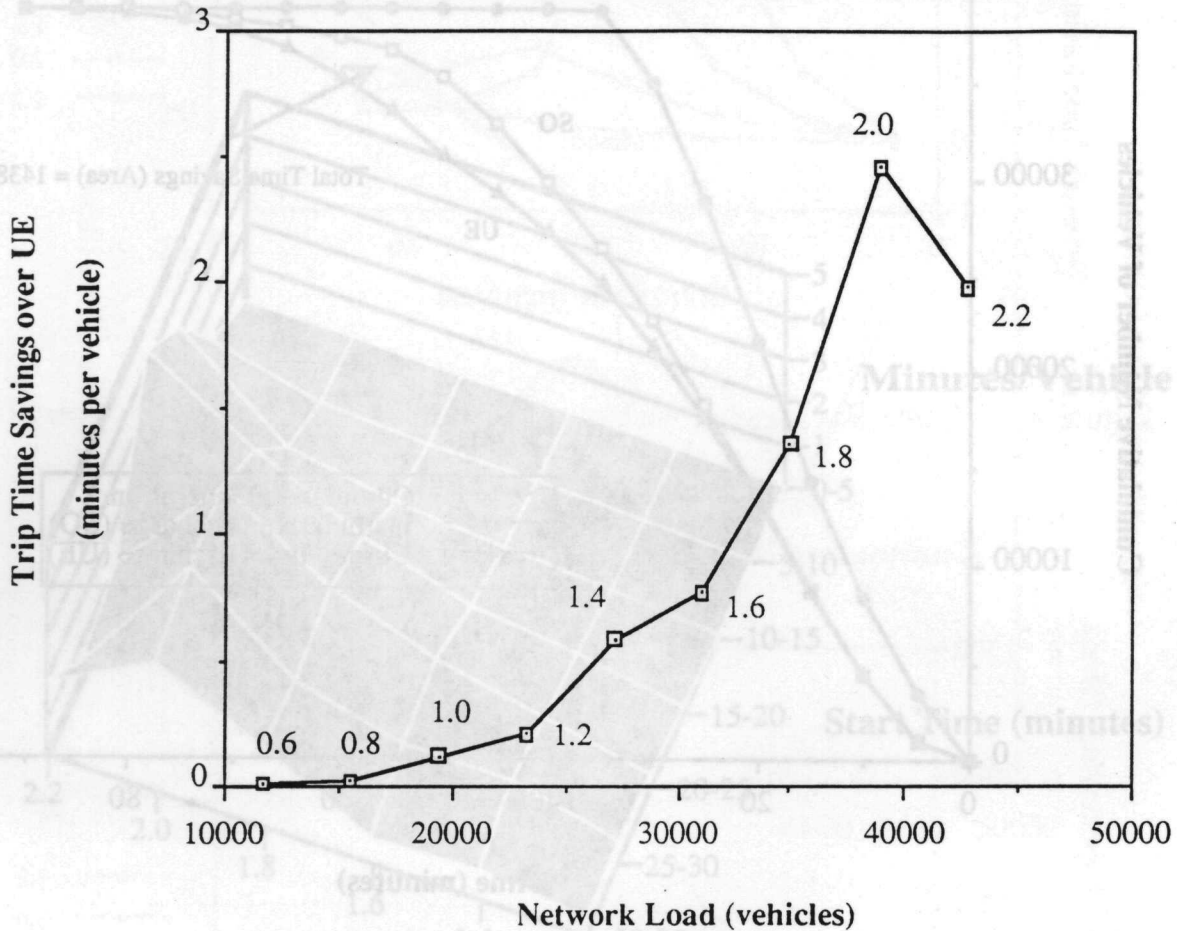
Figure 5. Trip Time Savings for SO over UE (in minutes/vehicle) as a Function of Network Load (the savings are assumed to be equally distributed among all the vehicles generated for that loading factor). The number by each plotted point is the corresponding loading factor.
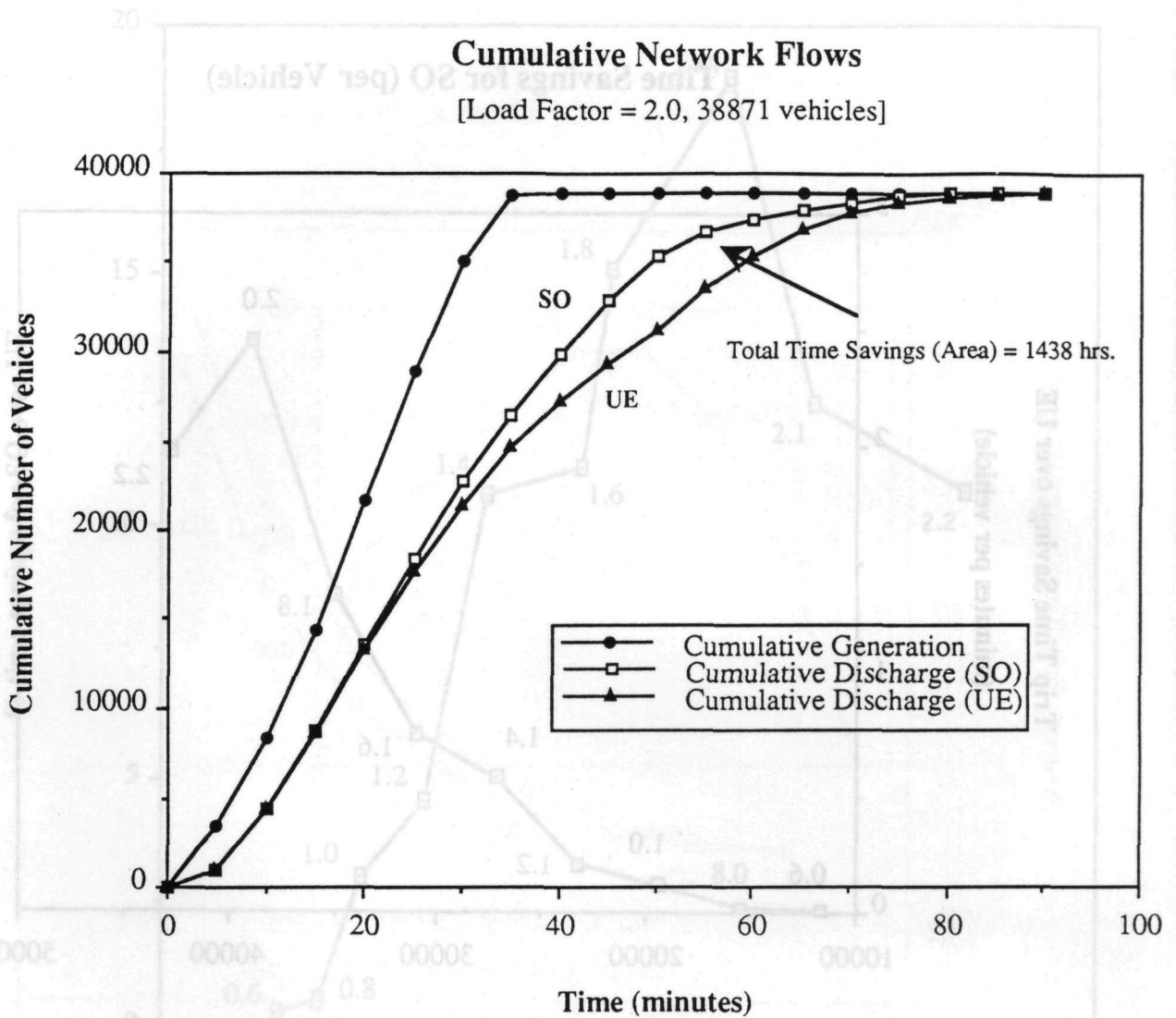
Cumulative Network Flows

[Load Factor = 2.0, 38871 vehicles]

SO

UE

Total Time Savings (Area) = 1438 hrs.

Cumulative Number of Vehicles

• Cumulative Generation
□ Cumulative Discharge (SO)
▲ Cumulative Discharge (UE)

Time (minutes)

Figure 6. Cumulative Generation Curve and SO and UE Cumulative Discharge Curves for a Loading Factor of 2.0. The points on the curve represent 5 minute updates of the cumulative number of vehicles. The area between the SO and UE discharge curves represents the time savings for SO over UE.

System Optimal Time Savings
(Relative to User Equilibrium)

Minutes/Vehicle

5
4
3
2
1
0-5
5-10
10-15
15-20   **Start Time (minutes)**
20-25
25-30
30-35

2.2
   2.0
      1.8
         1.6
            1.4
               1.2
Loading Factor
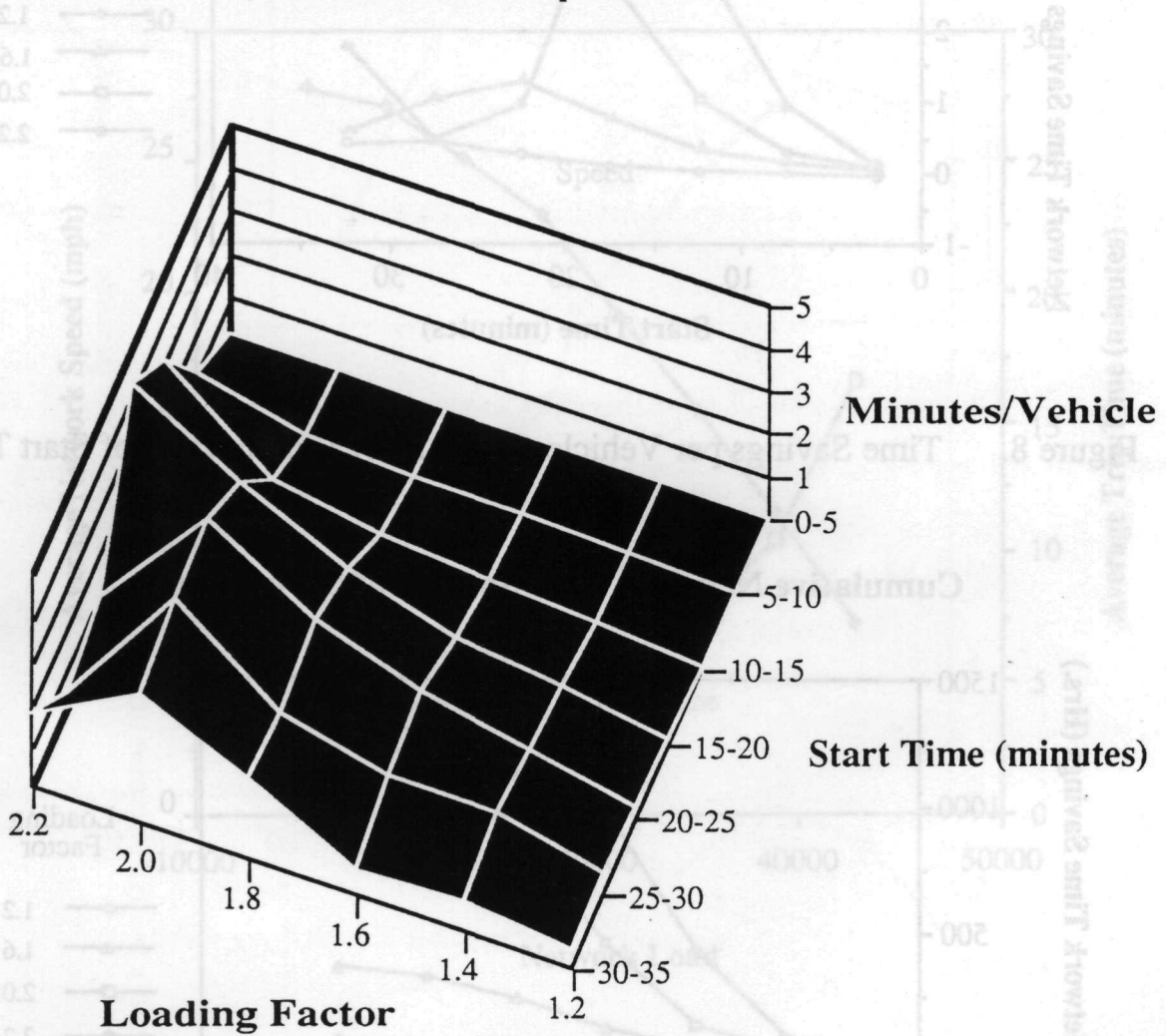
Figure 7.   Trip Time Savings (of SO relative to UE) per Vehicle (in minutes)
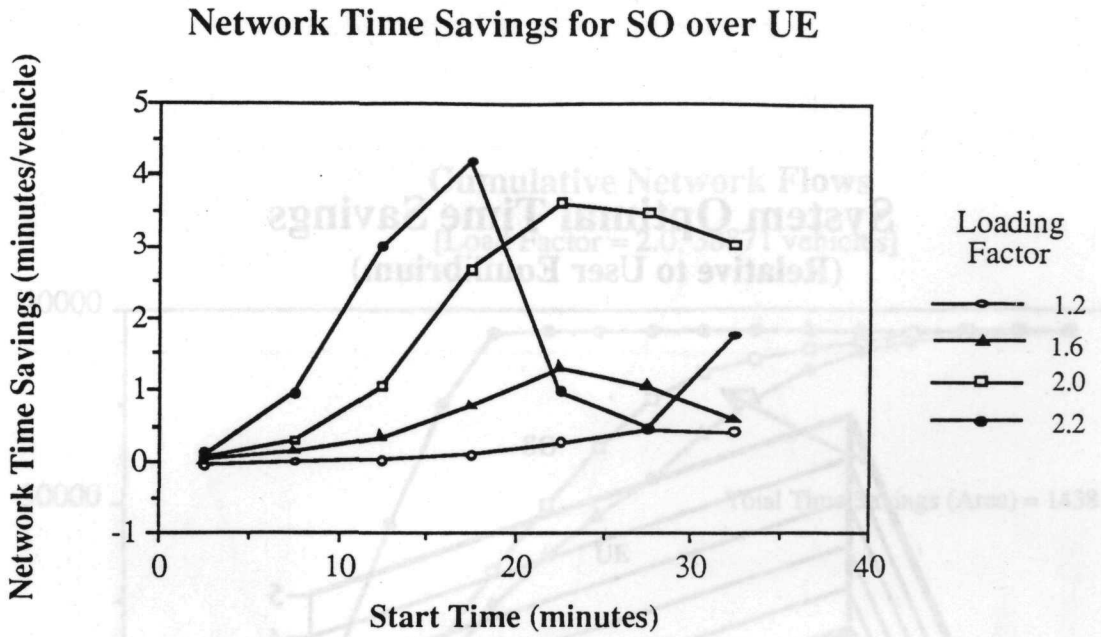as a Function of Loading Factor and Start Times (in minutes) of Vehicles

Network Time Savings for SO over UE

Figure 8.   Time Savings per Vehicle (in minutes) as a Function of Start Time

Cumulative Network Time Savings of SO over UE
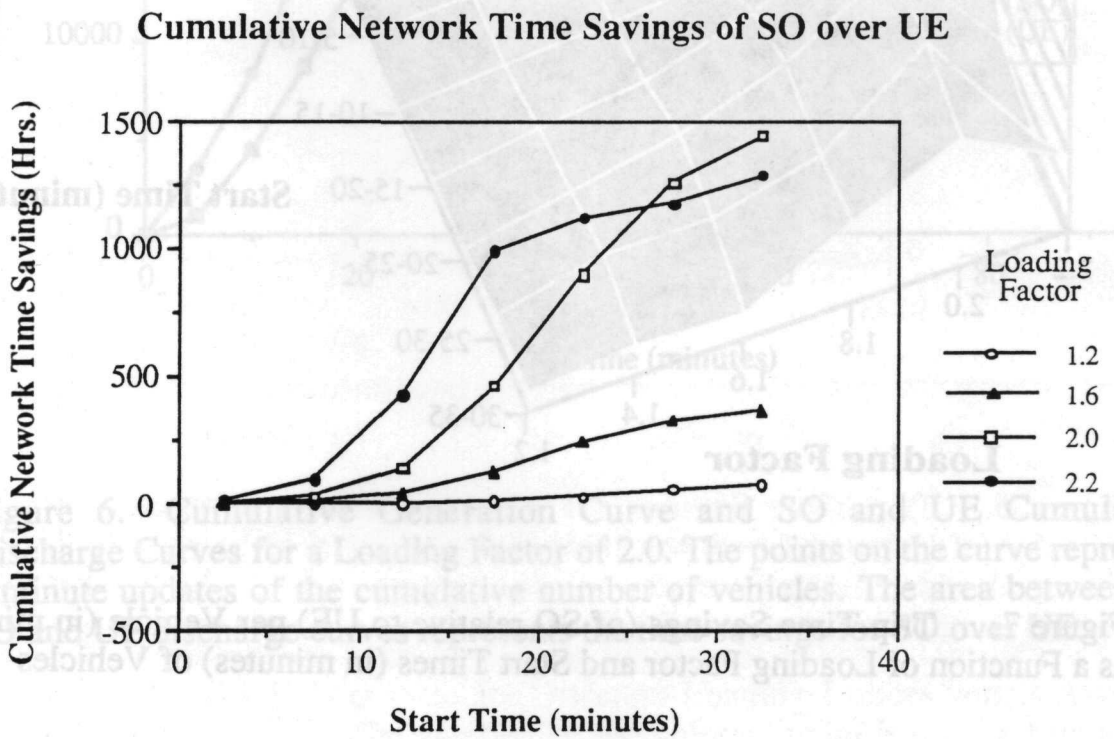
Figure 9.   Total Travel Time Savings (in Hours) as a Function of Start Time

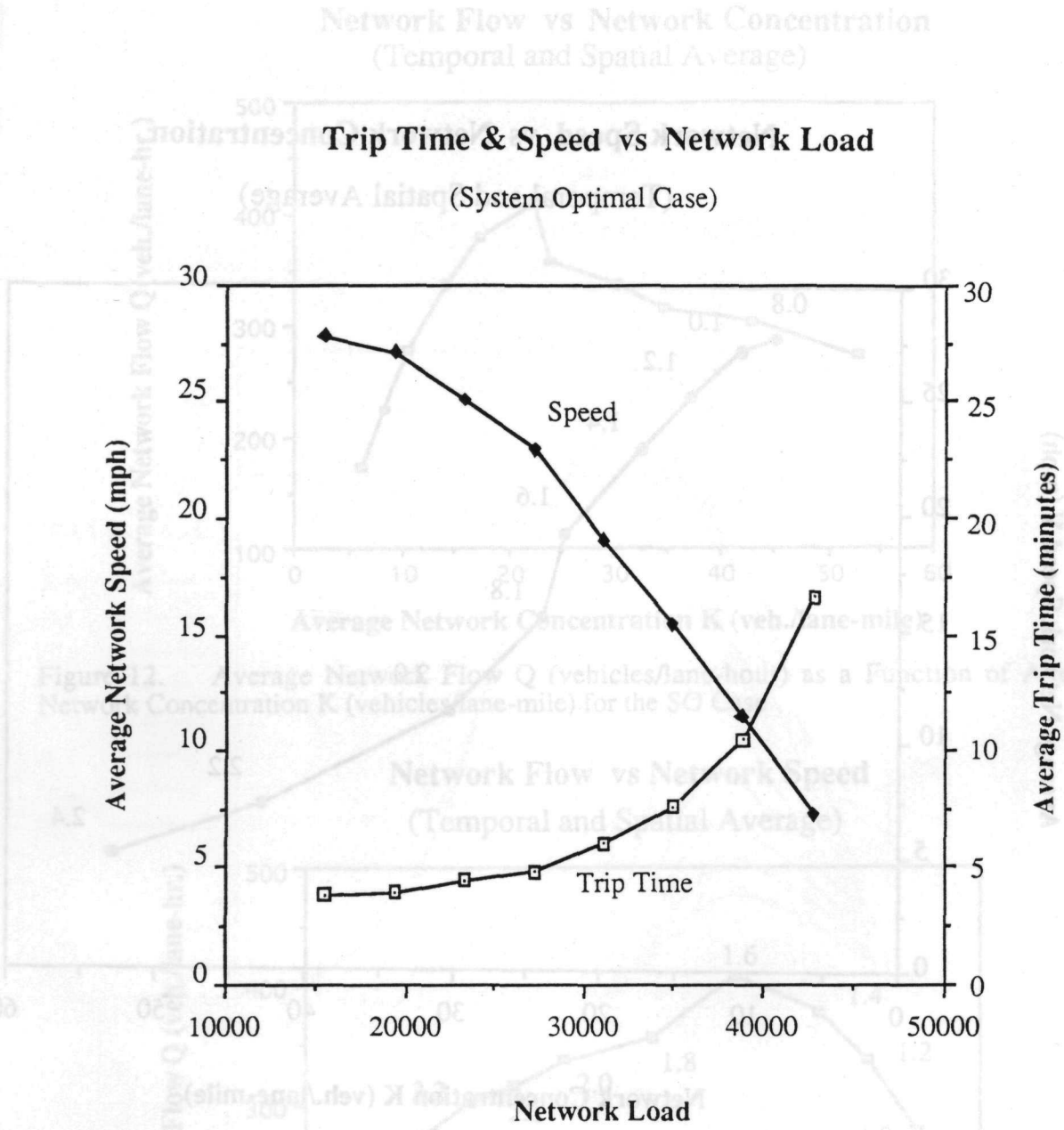**Trip Time & Speed vs Network Load**

(System Optimal Case)

Figure 10. Average Network Speed (mph) and Average Trip Time (minutes) for the System Optimal Case as a Function of Network Load (in number of vehicles)

Network Speed vs Network Concentration

(Temporal and Spatial Average)

Average Network Speed V (mph)

0.8

1.0

1.2

1.4

1.6

1.8

2.0

2.2

2.4

Network Concentration K (veh./lane-mile)

Figure 11. Average Network Speed V (mph) as a Function of Average
Network Concentration K (vehicles/lane-mile) for the System Optimal Case

Network Flow vs Network Concentration
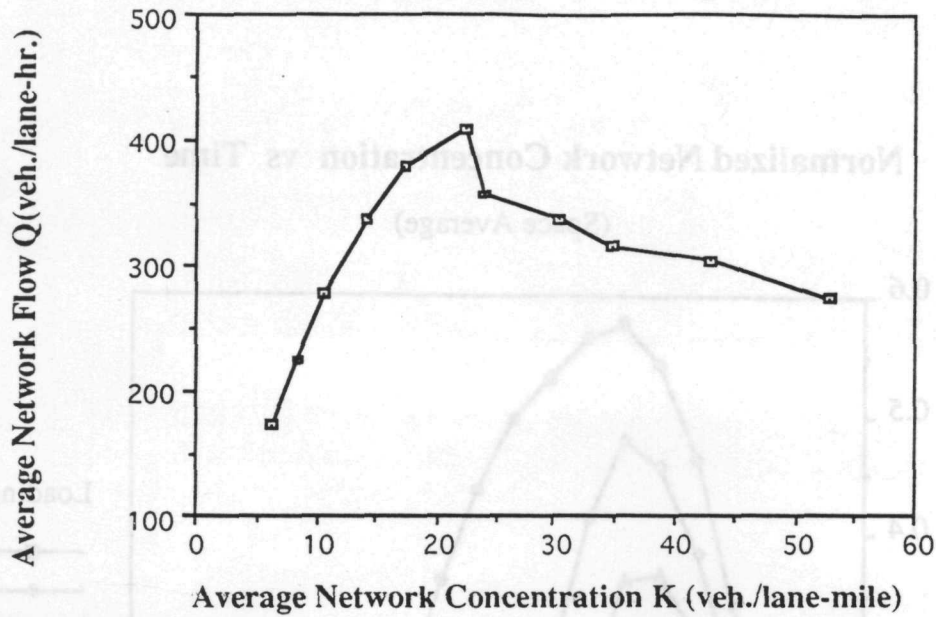### (Temporal and Spatial Average)

Figure 12.   Average Network Flow Q (vehicles/lane-hour) as a Function of Average Network Concentration K (vehicles/lane-mile) for the SO Case

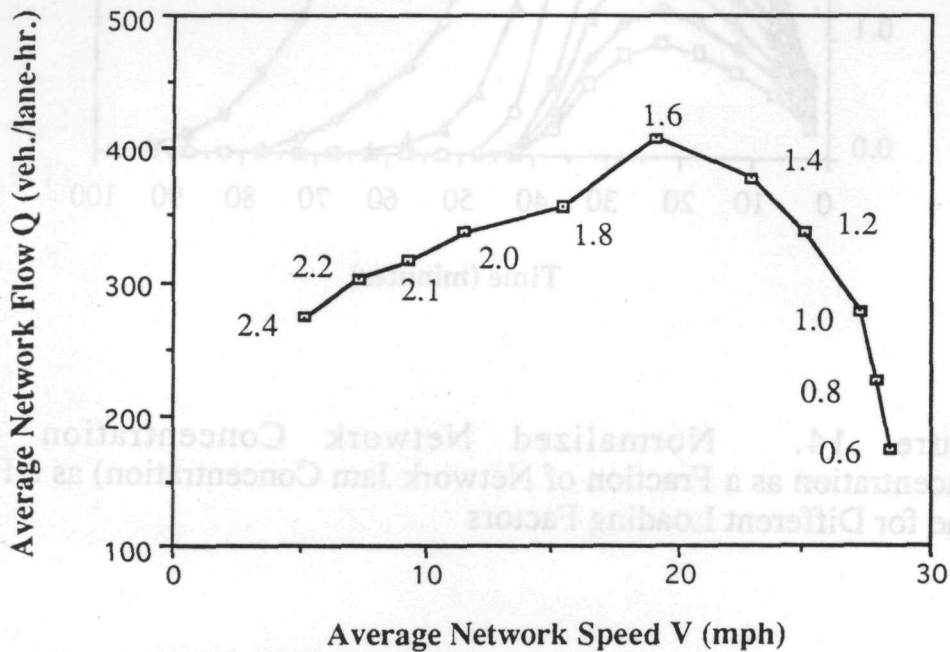## Network Flow vs Network Speed
(Temporal and Spatial Average)

Figure 13.   Average Network Flow Q (vehicles/lane-hour) as a Function of Average Network Speed (mph) for the SO Case

Normalized Network Concentration vs Time
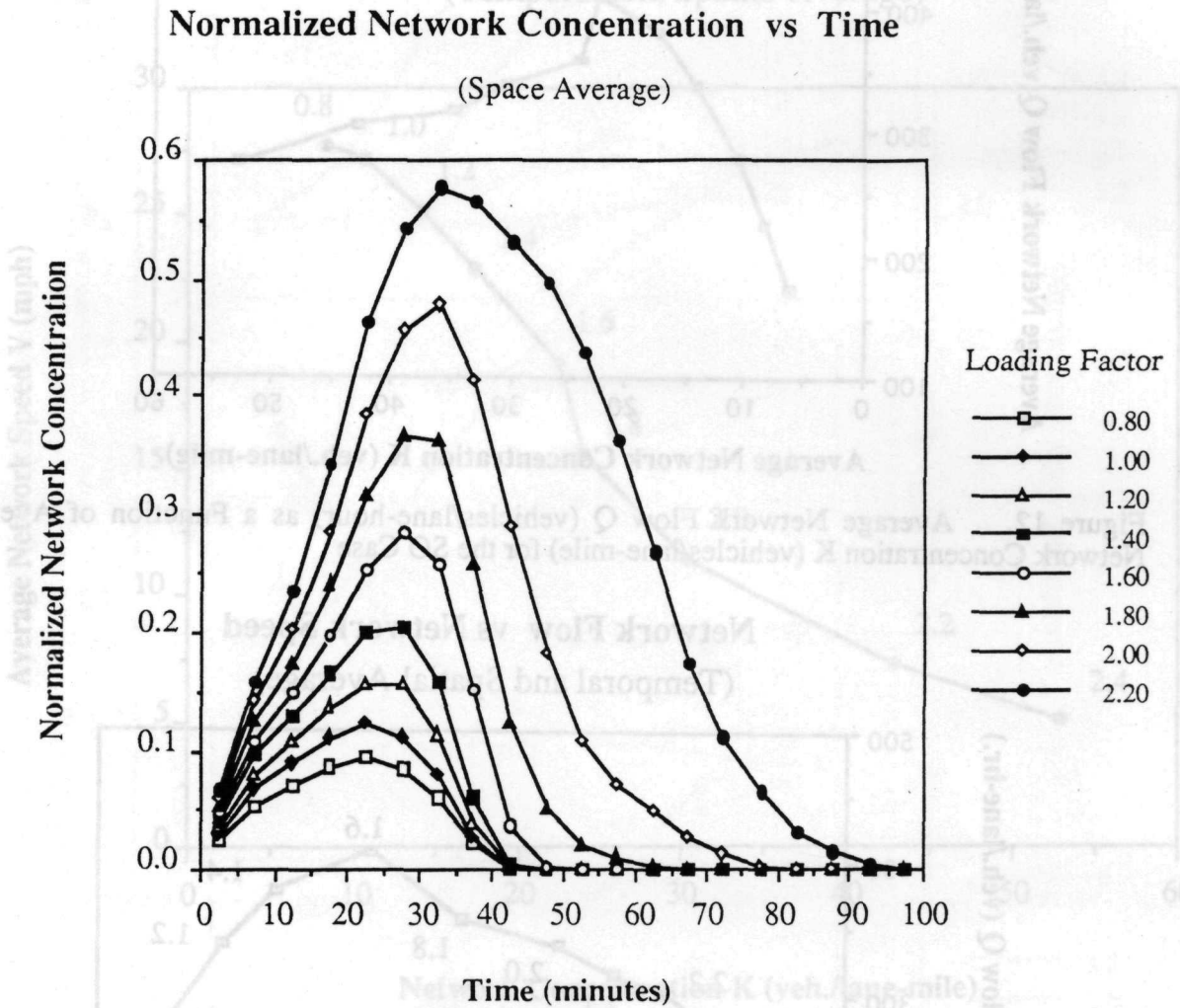
(Space Average)

Figure 14. Normalized Network Concentration (Network Concentration as a Fraction of Network Jam Concentration) as a Function of Time for Different Loading Factors